Exercises with Mother Daughter Relations and Equilibrium – Suggested Solutions

1) One kg of fresh and dry thorium nitrate, $Th(NO_3)_4$, is produced from a natural occurring thorium containing ore.

1. **Calculate the rate ^{232}Th and ^{228}Th disintegrates, and explain why the rate of disintegration is approximately equal for both of the thorium isotopes.**
   1000g $Th(NO_3)_4$ is 2.083 mol ⟶ N(Th) = $1.25*10^{24}$ atoms. This is natural thorium, where the equilibrium in Th-series will lead to equal activity of ^{232}Th and ^{228}Th. Since ^{232}Th has a incredibly long half-life and ^{228}Th is short compared to this and we can approximate N(Th)≈N(^{232}Th) = $1.25*10^{24}$. The disintegration for both is $1.96*10^{6}$ Bq.
2. **What is the weight of ^{228}Th in the thorium nitrate?**
   $6.46*10^{-8}$ g
3. **^{228}Ra separated from a one-year old thorium nitrate can be placed on a column to make an isotope generator. The chemical yield for this process is 90%. How much amount of thorium nitrate is required to make the column, if ^{228}Ra disintegrates with 10 000Bq?**
   10000 Bq ^{228}Ra = $2.62*10^{12}$ atoms = 90% ⟶ 100% $2.91*10^{12}$atoms. If ^{232}Th is N_1 and ^{228}Ra is N_2 we can use the formulas for mother/daughter relations:

   $$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} \times N_{1,0} \times (e^{\lambda_1 t} - e^{-\lambda_2 t})$$

   $$N_{1,0} = N_2 \times \frac{\lambda_2 - \lambda_1}{\lambda_1} \times \frac{1}{(e^{\lambda_1 t} - e^{-\lambda_2 t})} = 6.25 \times 10^{22} atoms$$

   $$\frac{6.26 \times 10^{22}}{6.022 \times 10^{23}} = 0.104\, mol\ \times 480.06 \frac{g}{mol} = 50\ g\ Th(NO_3)_4$$

   Iteratively it can be solved by using D(^{228}Ra) = 11 111 Bq:

   $$D_2 = D_1(1 - \frac{1}{2}^{t/T_{(1/2)}}) \rightarrow D_1 = \frac{D_2}{1 - \frac{1}{2}^{t/T_{(1/2)}}} = \frac{11111\,Bq}{1 - \frac{1}{2}^{1y/5.75y}} = 97838\,Bq$$
4. **At which rate will ^{224}Ra disintegrate 3 days after the column is made?**
   ^{228}Ra is formed from ^{228}Th, an immeasurable amount of ^{228}Th is formed in three days, and formation of new ^{224} Ra can therefore be ignored. D_0(^{224}Ra) = D_0(^{228}Th) = $1.36*10^{6}$ Bq. And we get a normal decay:
   $D = D_0 \times e^{-\lambda t} = 1.1 \times 10^{6} Bq$
5. **Are there any other radionuclides present at a noteworthy amount (more than 100Bq)?**
   ^{228}Ac, ^{220}Rn, ^{216}Po, ^{212}Pb, ^{212}Bi, ^{212}Po.

2) One gram of natural uranium is purified to contain no other elements than uranium at the time t equals 0.

1. **Which radionuclides are present at t=0?**
   When t=0 only the natural isotopes of uranium is present: ^{238}U,^{235}U and ^{234}U.
2. **That are their respective rates of disintegration?**
   D(^{238}U) = D(^{234}U) ≈ 12.5 kBq, D(^{235}U) = 575 Bq.
3. **Which radionuclides are present at t=25.5h?**
   When t = 23.5 h there is created some ^{234}Th and some ^{234}Pa, but creation of other daughters from ^{238}U is negligible. From the ^{235}U there is created ^{231}Th
4. **What are their respective rates of disintegration?**
   D(^{238}U) = D(^{234}U) =D_0,> D(^{234}Th) = D(^{234}Pa) = 376 Bq,
   D(235 U) = D0,
   D(231Th) = 287.5 Bq.
5. **Which radionuclides are present at t=24d?**
   When t = 24 days the same radionuclides are present.
6. **What are their respective rates of disintegration?**
   D(^{238}U) = D(^{234}U) =D_0,
   D(^{234}Th) = D(^{234}Pa) = 6250 Bq,
   D(^{235}U) = D_0,
   D(^{231}Th) = D(^{235}U) =575 Bq.
7. **Which radionuclides are present at t=1.0a?**
   When t = 1.0 y the same radionuclides are present.
8. **What are their respective rates of disintegration?**
   D(^{238}U) = D(^{234}U) =D_0,
   D(^{234}Th) = D(^{234}Pa) = D(^{238}U) = 12,5 kBq,
   D(^{235}U) = D_0,
   D(^{231}Th) = D(^{235}U) =575 Bq.
9. **Which radionuclides are present at t=10.0a?**
   When t = 10.0 y the same radionuclides are present.
10. **What are their respective rates of disintegration?**
    Same as 8.

3) A given amount of alum shale contains 10 gram naturally occurring uranium.

1. **Calculate the amount of Ra in Bq and in grams.**
   The shale contains all of the daughter products from ^{238}U and ^{235}U in equilibrium. In 10 g natural Uranium there is 125 kBq ^{238}U and 5.75 kBq ^{235}U which gives:
   $$D_{(^{226}Ra)} = D_{(^{238}U)} = 125\ kBq\ \rightarrow N = \frac{D}{\lambda} = 9.1 \times 10^{15} = 3.4 \times 10^{-6} g$$
   $$D_{(^{223}Ra)} = D_{(^{235}U)} = 5.75\ kBq\ \rightarrow N = \frac{D}{\lambda} = 8.2 \times 10^{9} = 3.0 \times 10^{-12} g$$
2. **Calculate the amount of Pb in grams and in Bq.**
   ^{210}Pb exist as a daughter from ^{238}U:
   $$D_{(^{210}Pb)} = D_{(^{238}U)} = 125\ kBq\ \rightarrow N = \frac{D}{\lambda} = 1.3 \times 10^{14} = 4.4 \times 10^{-8} g$$

3. **^{210}Pb is a weak beta emitter, but is still very toxic in large amounts. Why is it so?**
   One of the daughters is ^{210}Po, which is a alpha emitter and can do great harm inside the body. In addition, Pb is a daughter of radon which makes it possible for it to enter the lungs.

4) Some radionuclei have a major application in medicine. For practical reasons it is advantageous if these can be produced with the help of an isotope generator.

1. **Explain how an isotope generator may impact practical conditions.**
   It is easily accessible; it only needs to be processed once to create several doses of medicine.
2. **Which of the following medical radionuclides can be produced by an isotope generator? ^{67}Ga, ^{68}Ga, ^{64}Cu, ^{82}Br, ^{89}Sr, ^{90}Y, ^{123}I, ^{131}I, ^{177}Lu, ^{201}Tl, ^{211}At, ^{212}Pb, ^{211}At, ^{223}Ra.**
   The following nuclides can be extracted from a nuclide generator: ^{68}Ga, ^{90}Y, ^{212}Pb.
3. **In heart examination ^{201}Tl is used in amounts of 100MBq by intravenously injection of the radionuclide. Calculate the amount in mole and in grams?**
   100 MBq $^{201}Tl = 3.79*10^{13}$ atoms → m = $1.27*10^{-8}$ g.
4. **Thallium is a potent rat toxin, why do patients survive these examinations?**
   The amount inserted is too small to be considered poisonous for humans.

5) A daughter nuclide is eluted from a 100 MBq ^{44}Ti source in a column.

1. **How long does it take before 50 MBq of the daughter can be eluted from the system?**
   After one half-life it will be 50 MBq, which is 3.92 h (^{44}Sc).
2. **How many grams are 50 MBq of the daughter?**
   $D_1(^{44}Ti) \approx D_0(^{44}Ti)$ = 100 MBq, m(^{44}Ti) = $2.01*10^{-5}$ g
   $D_1(^{44}Sc)$ = 50 MBq, m(^{44}Sc) = $7.4*10^{-11}$ g.

6) Minerals containing thorium emits strong gamma radiation, without any of the involved thorium isotopes emitting gamma.

1. **Explain where the gamma radiation is coming from.**
   The gamma radiation comes from the daughters ^{228}Ac and ^{208}Tl.
2. **Which thorium isotopes exists in natural thorium, and what is the relation between them?**
   There are 2 radioisotopes in natural thorium, namely $^{232/228}Th$. They are in a secular equilibrium and the activity of the latter will be equal to the activity of the former. It is about $7.34*10^{9}$more mass of ^{232}Th than ^{228}Th.
3. **As a rule of thumb, it is said equilibrium is achieved after 10 half-lives. How much time must pass before equilibrium is reached throughout the entire chain of natural occurring thorium and uranium?**
   To achieve equilibrium through the whole series it needs to have taken ten times longer than the most long-lived daughter; for ^{238}U this is $2.455 \bullet 10^{6}$y for ^{235}U it is $3.276 \bullet 10^{5}$y and for ^{232}Th 57.5 y.

7) 100 grams of fresh metallic thorium is prepared from naturally occurring thorium containing minerals.

1. **What is the rate of disintegration?**
   100 g natural Th is more or less only ^{232}Th, this gives a decay of 405.9 kBq. D(^{229}Th) = D(^{232}Th), D(total) = 811.8 kBq.
2. **How many grams of ^{228}Th are present?**
   405.9 kBq is $1.34*10^{-8}$g.
3. **How many grams of ^{229}Th are present?**
   Nothing. ^{229}Th does not exist in nature.
4. **What is the total activity of the metallic thorium 7.2 days after it was made, and how is the branching ratio between alpha and beta radiation?**
   After 7.2 days the activity of ^{228}Ra will be 963 Bq and it will be in equilibrium with ^{228}Ac. ^{224}Ra is formed from ^{228}Th and after two half-lives there will be 75% of max possible ^{224}Ra in equilibrium with ^{210}Rn, ^{216}Po, ^{212}Pb, ^{212}Bi (assume 50% branching to ^{208}Tl and ^{212}Po). Total alpha activity: 2.2029 MBq, beta-activity: 610.7 kBq.

8) A source of 10 000 Bq from the very long lived ^{194}Hg is used to produce ^{194}Au. Assume 100% efficiency in the production process.

1. **How long does it take until 1000 Bq of the daughter can be extracted from the mother?**
   5.78 hours.
2. **How long does it take until 7500 Bq can be extracted?**
   After two half-lives, 76 hours, there will be 75%, 7500 Bq.

9) The radionuclide ^{211}At is planned to be used for medical purposes. It has a branching of 58% EC and 42% α.

1. **Explain why you get a long lived daughter from one of these branches.**
   The alpha decay of ^{211}At gives ^{207}Bi, with a half-life of 31.55 years.
2. **What is the weight of At in a 50 MBq ^{211}At sample?**
   $6.57*10^{-7}$ g.
3. **What is the rate of disintegration one week after the sample was made?**
   After one week all of the ^{211}At will decay to ^{207}Bi. The half-life is long enough (31.55 years) to do the approximation $N(^{207}Bi) \approx N_0(^{211}At)=1.87*10^{15}$.